Initialize R by entering the following commands at the prompt:

```r
options(contrasts=c("contr.sum","contr.poly"),digits=4) # set definition of contrasts
source(file=url("http://psycserv.mcmaster.ca/bennett/psy710/problems/ps5/p5init.R"))
```

1. An experiment was conducted to measure the effect of practice on visual contrast sensitivity. Detection threshold was measured on four consecutive days for eight subjects. The dependent variable was contrast sensitivity, which was defined as the reciprocal of the minimum amount of contrast needed to detect the pattern; higher sensitivity corresponds to better performance. The data are stored in the data frame `dat1`. Each row contains the data from one subject; the four columns contain contrast sensitivity measured on four days.

(a) Evaluate the effect of `day` with a univariate ANOVA. Use the Huynh-Feldt correction for all tests that make the sphericity assumption.

```r
names(dat1)
## [1] "day1" "day2" "day3" "day4"

day <- as.factor(c(1,2,3,4))
idata1 <- data.frame(day)
day.lm.01 <- lm(cbind(day1,day2,day3,day4)~1,data=dat1)
day.aov.01 <- Anova(day.lm.01,idata=idata1,idesign=~day,type="III")
summary(day.aov.01,multivariate=FALSE)
```

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<thead>
<tr>
<th></th>
<th>SS</th>
<th>num</th>
<th>Df</th>
<th>Error SS</th>
<th>den</th>
<th>Df</th>
<th>F</th>
<th>Pr(&gt;F)</th>
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</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>320359</td>
<td>1</td>
<td>5307</td>
<td>7 422.53</td>
<td>1.6e-07</td>
<td>***</td>
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<tr>
<td>day</td>
<td>1093</td>
<td>3</td>
<td>2616</td>
<td>21</td>
<td>2.93</td>
<td>0.058 .</td>
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<tr>
<td>Signif. codes:</td>
<td>0 '<em><strong>' 0.001 '</strong>' 0.01 '</em>' 0.05 '.' 0.1 ' ' 1</td>
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<td>Mauchly Tests for Sphericity</td>
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<tr>
<td>Test statistic p-value</td>
<td>0.251</td>
<td>0.167</td>
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<tr>
<td>Greenhouse-Geisser and Huynh-Feldt Corrections for Departure from Sphericity</td>
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<tr>
<td>GG eps Pr(&gt;F[GG])</td>
<td>0.538</td>
<td>0.1</td>
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<tr>
<td>HF eps Pr(&gt;F[HF])</td>
<td>0.6745</td>
<td>0.08591</td>
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</tbody>
</table>

**Answer:** The effect of day was not significant ($F(3,21) = 2.93$, $\tilde{e} = 0.675$, $p_{adjusted} = 0.086$).
The command `contr.poly` can be used to calculate the contrast weights to evaluate the linear and quadratic trends of contrast sensitivity across four evenly-spaced days:

```
contr.poly(n=4)
```

```
## [1,] -0.6708  0.5  -0.2236
## [2,] -0.2236 -0.5   0.6708
## [3,]  0.2236 -0.5  -0.6708
## [4,]  0.6708  0.5  -0.2236
```

The linear weights are in column 1 and the quadratic weights are in column 2. Use those weights to evaluate the linear and quadratic trends of contrast sensitivity across days.

```
lin.weights <- contr.poly(n=4)[,1]
```

```
## [1] -0.6708 -0.2236  0.2236  0.6708
```

```
quad.weights <- contr.poly(n=4)[,2]
```

```
## [1]  0.5  -0.5  -0.5  0.5
```

```
lin.scores <- as.matrix(dat1) %*% lin.weights
t.test(lin.scores)
```

```
## One Sample t-test
##
## data:  lin.scores
## t = 3.8, df = 7, p-value = 0.007
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  3.832 16.352
## sample estimates:
## mean of x
##  10.09
```

```
quad.scores <- as.matrix(dat1) %*% quad.weights
t.test(quad.scores)
```

```
## One Sample t-test
##
## data:  quad.scores
## t = -1, df = 7, p-value = 0.3
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## -17.794  6.973
## sample estimates:
## mean of x
## -5.411
```

**Answer:** The linear trend \( t(7) = 3.81, p < 0.01 \) differed significantly from zero, but the quadratic trend \( t(7) = -1.03, p = 0.33 \) did not. Assuming these were planned comparisons, I would reach the
same conclusion if I used the Bonferroni correction $p_{critical} = .05/2 = 0.025$ to maintain a familywise Type I error rate of 0.05.

2. An experiment was conducted to assess the effects of factors A and B on performance. The experiment used a 2 x 2 factorial within-subjects design: the two levels of factor A were crossed with the two levels of factor B, and each subject was tested in all four conditions in a random order. The data are stored in the data frame `dat2`. Evaluate the effects of A and B with a univariate ANOVA. If the A x B interaction is significant, then evaluate the simple main effect of B at each level of A.

```r
names(dat2)
## [1] "a1b1" "a1b2" "a2b1" "a2b2"

a <- as.factor(c("a1","a1","a2","a2"))
b <- as.factor(c("b1","b2","b1","b2"))
dat2.idata <- data.frame(a,b)
dat2.lm.01 <- lm(cbind(a1b1,a1b2,a2b1,a2b2)~1,data=dat2)
dat2.aov.01 <- Anova(dat2.lm.01, idata=dat2.idata, idesign=~a*b,type="III")
summary(dat2.aov.01,multivariate=FALSE)

## Univariate Type III Repeated-Measures ANOVA Assuming Sphericity
## (Intercept) 289733 1   3663 7 553.74 6.4e-08 ***
## a 138 1 690 7 1.40 0.276
## b 429 1 2325 7 1.29 0.293
## a:b 587 1 521 7 7.89 0.026 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# EVALUATE SIMPLE MAIN EFFECT OF B AT A1:
a1 <- dat2[,1:2]
b <- as.factor(c("b1","b2"))
dat2.lm.02 <- lm(cbind(a1b1,a1b2)~1,data=a1)
summary(Anova(dat2.lm.02,idata=data.frame(b),idesign="b,type="III"),multivariate=FALSE)

## Univariate Type III Repeated-Measures ANOVA Assuming Sphericity
## (Intercept) 151253 1   1638 7 646.28 3.7e-08 ***
## b 6 1 2055 7 0.02 0.89
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# EVALUATE SIMPLE MAIN EFFECT OF B AT A2:
a2 <- dat2[,3:4]
dat2.lm.03 <- lm(cbind(a2b1,a2b2)~1,data=a2)
summary(Anova(dat2.lm.03,idata=data.frame(b),idesign="b,type="III"),multivariate=FALSE)
```
3. An experiment measured memory for lists of words in young, middle-aged, and senior adults. The experiment used three sets of words (w1, w2, and w3) that varied in terms of complexity, and two study-test intervals (short and long). The three word sets and two study-test intervals were combined in a 3 (word) x 2 (delay) x 3 (group) split-plot design. The dependent variable was the score on a memory test; higher numbers indicate better performance. The data are stored in the data frame dat3.

(a) Conduct a split-plot ANOVA to evaluate the effects of word list, study-test interval, and age group (and all interactions) on memory. Where appropriate, adjust p-values with the Huynh-Feldt correction.

**Answer:** The ANOVA tables and corrected p-values are listed in the following output:

```r
names(dat3)
## [1] "w1.short" "w1.long" "w2.short" "w2.long" "w3.short" "w3.long" "group"

words <- as.factor(c("w1","w1","w2","w2","w3","w3"))
delay <- as.factor(c("short","long","short","long","short","long"))
dat3.idata <- data.frame(words,delay)
dat3.lm.01 <- lm(cbind(w1.short,w1.long,w2.short,w2.long,w3.short,w3.long)~group,data=dat3)
dat3.aov.01 <- Anova(dat3.lm.01, idata=dat3.idata, idesign=~words*delay,type="III")
summary(dat3.aov.01,multivariate=FALSE)
## Warning in summary.Anova.mlm(dat3.aov.01, multivariate = FALSE): HF eps > 1 treated as 1
##
## Univariate Type III Repeated-Measures ANOVA Assuming Sphericity
##
## SS num Df Error SS den Df F Pr(>F)
## (Intercept) 1429764 1 1568 21 19144.24 <2e-16 ***
## group 519 2 1568 21 3.48 0.050 *
## words 13 2 413 42 0.64 0.531
## group:words 47 4 413 42 1.19 0.331
## delay 852 1 204 21 87.81 6e-09 ***
## group:delay 74 2 204 21 3.79 0.039 *
## words:delay 57 2 336 42 3.56 0.037 *
## group:words:delay 54 4 336 42 1.70 0.169
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '. ' 0.1 ' ' 1
##
## Mauchly Tests for Sphericity
##
## Test statistic p-value
## words 0.817 0.132
## group:words 0.817 0.132
## words:delay 0.968 0.725
## group:words:delay 0.968 0.725
##
```
(b) The experimenter believes that the effect of test-study interval differs for low complexity ($w_1$) and high complexity ($w_3$) words, and that this difference may vary across age groups. Evaluate these hypotheses conducting a single ANOVA on a set of contrast scores.

```r
names(dat3)
## [1] "w1.short" "w1.long" "w2.short" "w2.long" "w3.short" "w3.long" "group"

y.mat <- as.matrix(dat3[,1:6])
c.weights <- c(-1,1,0,0,1,-1)
c.scores <- y.mat %*% c.weights
summary(aov(c.scores ~ 1 + dat3$group),intercept=TRUE)
## Df  Sum Sq Mean Sq F value Pr(>F)
## (Intercept) 1 224.1 224.1 7.66 0.012 *
## dat3$group 2  67.3 33.7  1.06 0.368
## Residuals 21  29.3  1.4
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

**Answer:** When we define effects using the sum-to-zero constraint, the intercept corresponds to the grand mean. In our ANOVA, the intercept differs significantly from zero ($F(1,21) = 7.66, p = 0.012$), and therefore we reject the null hypothesis that the difference between short and long test-study intervals is the same for low and high complexity words. The effect of `group` is not significant, so we do not reject the null hypothesis that the linear contrast varies across age groups (i.e., there is no contrast $\times$ group interaction).

4. An experiment was conducted to measure the effects of average luminance (low vs. high) and stimulus duration (100, 200, and 300 msec) on sensitivity for visual patterns in younger and older subjects. The two levels of luminance and three stimulus durations were combined factorially, and each subject was tested in all six conditions. The data are stored in the data frame `dat4`.

(a) The experimenter is interested in determining if there is a significant linear trend of sensitivity across stimulus duration, and if the trend varies as a function of luminance and/or age group. Evaluate these hypotheses by conducting a single, split-plot ANOVA. (N.B. The weights for a linear trend across 3 equally-spaced levels are `c(-1, 0, 1)`.)
names(dat4)
## [1] "d100.low"  "d200.low"  "d300.low"  "d100.high"  "d200.high"  "d300.high"
## [7] "group"

# extract data:
y.mat.low <- as.matrix(dat4[,1:3])
colnames(y.mat.low) <- names(dat4)[1:3]
y.mat.high <- as.matrix(dat4[,4:6])
colnames(y.mat.high) <- names(dat4)[4:6]
# compute linear trends:
lin.weights <- c(-1,0,1)
lin.scores.low <- y.mat.low %*% lin.weights
lin.scores.high <- y.mat.high %*% lin.weights
# combine trend scores:
lin.scores <- cbind(lin.scores.low,lin.scores.high)
colnames(lin.scores) <- c("lin.low","lin.high")
# perform statistical test:
lum <- as.factor(c("low","high"))
lin.idata <- data.frame(lum)
lin.mlm.01 <- lm(lin.scores ~ 1 + dat4$group)
lin.aov.01 <- Anova(lin.mlm.01,idata=lin.idata,idesign=~lum,type="III")
summary(lin.aov.01, multivariate=FALSE)

## Univariate Type III Repeated-Measures ANOVA Assuming Sphericity
##
##              SS num Df Error SS den Df  F    Pr(>F)
## (Intercept)   6723  1     187   14 504.02 2.2e-12 ***
## dat4$group   755  1     187   14  56.60 2.8e-06 ***
## lum           2   1     211   14  0.10     0.75
## dat4$group:lum 29   1     211   14  1.95     0.18
##---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

**Answer:** When we define effects using the sum-to-zero constraint, the intercept corresponds to the grand mean. In our ANOVA, the intercept differs significantly from zero \( (F(1,187) = 504, p < .0002) \), and therefore we reject the null hypothesis that the linear trend is zero. The effect of `group` is significant, so we reject the null hypothesis that the linear trend does not differ across age groups (i.e., there is trend \( \times \) group interaction). There is no effect of luminance, so the linear trend did not differ as a function of luminance. Finally, the group \( \times \) luminance interaction was not significant (i.e, there was no trend \( \times \) group \( \times \) luminance interaction), which implies that the trend \( \times \) group interaction did not depend on luminance.

(b) What is the sphericity assumption? Would the sphericity assumption have any impact on your interpretation of the results of the previous analysis? Explain

**Answer:** We will discuss the answer to this question in class.