Statistics Lab
One-way Within-Subject ANOVA

PSYCH 710

9 One-way Within-Subjects ANOVA

Section 9.1 reviews the basic commands you need to perform a one-way, within-subject ANOVA and to evaluate a linear trend/contrast. Lab questions are listed in Section 9.2.

9.1 Review of example from Class Notes

In this section we will work through the example given in the course notes for Chapter 11. First, initialize R by entering the following commands at the prompt. You must type the commands exactly as shown.

```r
options(contrasts=c("contr.sum","contr.poly") )
library(car)
mw115 <- read.csv(file=url("http://psycserv.mcmaster.ca/bennett/psy710/datasets/mw115.csv"))
```

The last command reads a csv file that contains the data from Table 11.5 in your textbook, and stores everything in the data frame mw115. The data are from a fictitious experiment that measured cognitive ability in 12 children at 30, 36, 42, and 48 months of age.

The within-subjects ANOVA consists of the following steps:

1. The dependent variables are extracted from the data frame and stored in a matrix;
2. The `lm` function is used to create a multivariate linear model (i.e., mlm) object that specifies the between-subjects aspect of the experimental design;
3. The `Anova` function in the car package is used to i) specify the within-subjects aspects of the design; and ii) convert the mlm object to an aov object;
4. Finally, the `summary` function prints the ANOVA table, Mauchly Test, and the corrected p values for the within-subject variables.

9.1.1 extracting the data

Note the wide format of the data frame. Each row contains all of the data from a single subject, and each column contains all of the data in a single condition.

```
mw115[1:4,] # print 1st 4 rows
```

```
##   subj age.30 age.36 age.42 age.48
## 1    s1    108     96    110    122
## 2    s2    103    117    127    133
## 3    s3     96    107    106    107
## 4    s4     84     85     92     99
```

To do the analysis, we first extract the dependent variables and create a matrix. Note how a data frame and a matrix are different classes of variables:
We can calculate summary statistics for our dependent variables using the `apply` function:

```r
which.dim <- 2 # 1=rows; 2=columns
apply(y.mat,2,mean) # column means
## age.30  age.36  age.42  age.48
## 108  103   96  100  110  117   96  107  106  107
apply(y.mat,2,sd) # column standard dev
## age.30  age.36  age.42  age.48
apply(y.mat,2,length)
## age.30  age.36  age.42  age.48
## 12  12  12  12
# summary(y.mat) # try this command
```

### 9.1.2 multivariate linear models

Next, we use `lm` to create a multivariate linear model:

```r
mw115.mlm <- lm(y.mat ~ 1) # multivariate linear model
```

Note that the model contains multiple dependent variables and only an intercept (1) on the side of the formula that contains the independent (or predictor) variables. If our experiment had contained between-subject factors, they would have been included, too. However, there are no between-subject factors and so that side of the formula contains only the intercept.
9.1.3 specifying the within-subjects part of the design

Next, we construct a data frame that specifies the factors used in the within-subject part of the design. In this case there is only one factor, \textit{age}. The following code creates the factor and stores it in a data frame:

\begin{verbatim}
(age <- factor(x=c("a30","a36","a42","a48")) )
## [1] a30 a36 a42 a48
## Levels: a30 a36 a42 a48
(mw115.idata <- data.frame(age))
## age
## 1 a30
## 2 a36
## 3 a42
## 4 a48
\end{verbatim}

It is important to see that the order of the levels of \textit{age} (from top to bottom) match the order of levels in \textit{y.mat} (from left to right). In other words, the factor \textit{age} can be used as labels for each column in the dependent variable matrix.

Finally, we use the \texttt{Anova} command in the \texttt{car} package to construct an \texttt{aov} object.

\begin{verbatim}
library(car)
mw115.aov <- Anova(mw115.mlm,idata=mw115.idata,idesign=~age,type="III")
\end{verbatim}

In the \texttt{Anova} command, the \texttt{idata} parameter is the data frame that contains all of the within-subject factors in our experiment. The \texttt{idesign} parameter is a \texttt{one-sided} formula that specifies how these factors are used in the within-subject model. In this case, the model contains only one factor, \textit{age}, and therefore the model is very simple. Finally, Type III sums of squares are specified only to suppress a warning message produce by \texttt{Anova} when type is not specified: it does not make a difference in this case because the design is balanced.

9.1.4 printing the ANOVA table

The \texttt{summary} function prints the ANOVA table. The default is to print various multivariate statistics; setting \texttt{multivariate} to \texttt{FALSE} suppresses that behaviour:

\begin{verbatim}
summary(mw115.aov,multivariate=FALSE)
##
## Univariate Type III Repeated-Measures ANOVA Assuming Sphericity
##
## Sum Sq num Df Error SS den Df F value Pr(>F)
## (Intercept) 559872 1 6624 11 929.74 5.6e-12 ***
## age 552 3 2006 33 3.03 0.043 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Mauchly Tests for Sphericity
##
## Test statistic p-value
## age 0.243 0.0177
##
## Greenhouse-Geisser and Huynh-Feldt Corrections
\end{verbatim}
## for Departure from Sphericity
##
## GG eps Pr(>F[GG])
## age 0.61 0.075 .
##---
## Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
##
## HF eps Pr(>F[HF])
## age 0.72485 0.063538

The print out includes the Geisser-Greenhouse and Huyhn-Feldt adjustments, as well as the unadjusted $F$ test. The **Mauchly Test** of sphericity can be used to see if the deviations from sphericity are significant. If the test is not significant, then the unadjusted $F$ test may be appropriate. Some have argued that the Mauchly Test lacks power, and so you might want to use a liberal criterion (e.g., $p < 0.1$ or $p < 0.2$) for rejecting the null hypothesis (of having spherical data).

### 9.1.5 Geisser-Greenhouse conservative $F$ test

You can calculate a $p$ value using the **lower-bound adjustment** with the `pf` command. The numerator and denominator degrees of freedom are, respectively, 1 and one less than the number of subjects.

```r
dim(y.mat)
## [1] 12 4
n <- 12;
F <- 3.027  # from ANOVA
1-pf(F,df1=1,df2=n-1)  # not significant, \( F(1,11)=3.027, p=0.11 \)
## [1] 0.10975
```

### 9.1.6 linear contrasts/trends

The adjusted $F$ tests are not significant. However, the omnibus test lacks power and specificity. We probably are not interested in *any* difference among our dependent measures. Instead, we probably would be looking for a linear and/or quadratic changes, or trends, in the dependent variable across age. The `contr.poly` command can be used to get the weights that we need for our trend analysis: \( n \) specifies the number of levels in the factor and `scores` specifies the values of the levels. Note that `scores` is necessary only when the levels are not spaced evenly, however it does not hurt to use them here:

```r
( theWeights <- contr.poly(n=4,scores=c(30,36,42,48)) )
##
## [1,] -0.67082 0.5 -0.22361
## [2,] -0.22361 -0.5 0.67082
## [3,] 0.22361 -0.5 -0.67082
## [4,] 0.67082 0.5 0.22361

lin.weights <- theWeights[,1]  # save 1st column
(quad.weights <- -1* theWeights[,2])  # save 2nd column
## [1] -0.5 0.5 0.5 -0.5
```

Notice that I multiplied the quadratic weights by -1. Why? Because I wanted the weights to match the direction of the trend that I think I will find in my data (i.e., increasing and then decreasing).
Next, we use the trend weights to convert the four measures for each subject into a single composite score: each measure is multiplied by the corresponding weight, and the four products are summed to create a single score:

\[
\text{lin.scores} \leftarrow y.\text{mat} \times \text{lin.weights} \\
\text{quad.scores} \leftarrow y.\text{mat} \times \text{quad.weights}
\]

Finally, for each trend we use a t tests to evaluate the null hypothesis that the trend is zero. The linear trend is significantly different from zero, but the quadratic trend is not:

\[
\begin{align*}
\text{t.test(lin.scores)} \\
\text{t.test(quad.scores)}
\end{align*}
\]

9.2 Lab Questions

9.2.1 ANOVA on trackbox data

The data for this section can be loaded into R with the following command:

\[
\text{load(url("http://psycserv.mcmaster.ca/bennett/psy710/datasets/trackbox09.Rdata"))}
\]

“Discovery Day” is a day set aside by the United States Naval Postgraduate School in Monterey, California, to invite the general public into its laboratories. On Discovery Day, 21 October 1995, data on reaction time and hand-eye coordination were collected on 118 members of the public who visited the Human Systems Integration Laboratory. The age and sex of each subject were also recorded. Visitors were mostly in family groups. A rotary pursuit tracking experiment was done to examine motor learning and hand-eye coordination. The equipment was a rotating disk with a 3/4” target spot. In the “Circle” condition, the target spot moved at a constant speed in a circular path. In the “Box” condition, the target spot moved at various speeds as it moved along a box-shaped path. The subject’s task was to maintain contact with the target spot with a metal wand. Four trials were recorded for each of 108 subjects. Each trial lasted 15 s, and the total contact duration during each trial was recorded. The data from the Box condition (n=70) are stored in track.data. The variables
are *sex, age, subject, time1, time2, time3, time4*. The dependent variable, the amount of time the subject maintained contact with the target on each trial, is in the variables *time1-4*.

In the following analyses, we will ignore *sex* and *age*.

1. Extract the four dependent variables (i.e., contact duration on each trial) and store the result in a matrix.

```r
names(track.data)
## [1] "sex"  "age"  "subject" "time1" "time2" "time3" "time4"

track.times <- as.matrix(track.data[,4:7])
class(track.times)
## [1] "matrix"
dim(track.times)
## [1] 70 4
```

2. Calculate the mean and standard deviation of contact time on each trial.

```r
apply(track.times,2,mean)
## time1 time2 time3 time4
## 2.5974 3.3931 3.6344 3.9020
apply(track.times,2,sd)
## time1 time2 time3 time4
## 1.7803 2.0286 2.1134 2.3281
```

3. Use a within-subjects (repeated-measures) ANOVA to determine if contact time differed across trials.

```r
(trial <- factor(x=c("t1","t2","t3","t4"),ordered=FALSE) )
## [1] t1 t2 t3 t4
## Levels: t1 t2 t3 t4

( track.idata <- data.frame(trial) )
## trial
## 1 t1
## 2 t2
## 3 t3
## 4 t4

track.mlm <- lm(track.times ~ 1)
track.aov<-Anova(track.mlm,idata=track.idata,idesign=~trial,type="III")
summary(track.aov,multivariate=FALSE)
```
(a) Adjust the $p$ value for the effect of trial with the **conservative**, or lower-bound, F test.

```r
length(track.data$subject)
## [1] 70
F.trial <- 40.767
n<-70
df1 <- 1
df2 <- (n-1)
1-pf(40.767,df1=df1,df2=df2)
## [1] 1.6991e-08
```

(b) Adjust the $p$ value for the effect of trial with the **Huynh-Feldt** corrections for sphericity.

**Answer:** The Mauchly test was significant, and therefore the sphericity assumption is not valid. Using the Geisser-Greenhouse correct, the effect of trial was significant ($F(3, 207) = 40.767, \hat{\varepsilon} = 0.87, p < .00001$). Also, the effect was significant if we use the Huynh-Feldt correction ($F(3, 207) = 40.767, \tilde{\varepsilon} = 0.91, p < .00001$).

4. Use a linear contrast to test for a positive (i.e., increasing) linear trend in contact time across trials.

**Answer:** The following commands create contrast weights, use them to convert our four dependent variables into a single composite score, and final perform a one-tailed $t$ test on the composite scores:

```r
# R can calculate trend weights:
contr.poly(n=4,scores=c(1,2,3,4))
```

```r
## .L .Q .C
## [1,] -0.67082 0.5 -0.22361
## [2,] -0.22361 -0.5 0.67082
```
9.2.2 chick weight data

In this section we will conduct use ANOVA and trend analysis to analyze a subset of R’s ChickWeight data set which contains body weights of four groups of chicks that were measured at birth and every second day until day 20. The data frame chicks contains weights measured from days 0 though 10 from one group of chicks:

```r
load(file=url("http://psycserv.mcmaster.ca/bennett/psy710/datasets/chicks.Rdata") )
closeAllConnections()
```

The data frame chicks contains a subset of the data in R’s ChickWeight data set, which contains body weights of four groups of chicks that were measured at birth and every second day until day 20. The data frame chicks, which contains weights measured from days 0 though 10 from one group of chicks, can be loaded with the following command:

1. Without looking at the data, do you think it is likely that the sphericity assumption is valid for this design? Explain.

   **Answer:** In all likelihood, no, the sphericity assumption is not reasonable for these data. Data satisfy the sphericity assumption if the variances of the difference scores between all possible pairs of dependent measures are equal. In this design, I would expect the difference between two measures to increase as a function of time between measures.

2. Create weights that can be used to test for linear and quadratic trends of weight across days. Verify that the two sets of weights are orthogonal.

   # levels of day are space equally, so we can use default contr.poly:
   contr.poly(n=6)

   ```
   ## [1,] -0.59761 0.54554 -0.37268 0.18898 -0.062994
   ## [2,] -0.35857 -0.10911 0.52175 -0.56695 0.314970
   ## [3,] -0.11952 -0.43644 0.29814 0.37796 -0.629941
   ## [4,] 0.11952 -0.43644 -0.29814 0.37796 0.629941
   ## [5,] 0.35857 -0.10911 -0.52175 -0.56695 -0.314970
   ## [6,] 0.59761 0.54554 0.37268 0.18898 0.062994
   ```
3. Evaluate the null hypothesis that the linear trend of weight across days is \( \leq 0 \).

```r
# IMPORTANT: we need to extract our data and save in a matrix:
chicks.mat <- as.matrix(chicks)
lin.scores <- chicks.mat %*% lin.weights;
t.test(lin.scores,alternative="greater")
```

##
## One Sample t-test
## data:  lin.scores
## t = 10.2, df = 18, p-value = 3.2e-09
## alternative hypothesis: true mean is greater than 0
## 95 percent confidence interval:
## 36.06  Inf
## sample estimates:
## mean of x
## 43.425

**Answer:** The linear trend of weight across days was significantly greater than zero \( (t(18) = 10.2, p < .00001, \text{one-tailed}) \).

4. Use the weights to evaluate the hypothesis that the quadratic trend of weight across days is zero.

```r
quad.scores <- chicks.mat %*% quad.weights;
t.test(quad.scores) # two-tailed test!
```

##
## One Sample t-test
## data:  quad.scores
## t = 3.26, df = 18, p-value = 0.0043
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## 2.0330 9.3717
## sample estimates:
## mean of x
## 5.7024

**Answer:** The quadratic trend of weight across days differed significantly from zero \( (t(18) = 3.26, p = .004) \).