Hypothesis Testing Example
Diagnosing Eye Disorders Based on Intra-ocular Pressure

• Unusually low or high intra-ocular pressure is associated with eye disease.
• Important clinical task: use pressure measure to identify eyes for follow-up tests

Distribution of Typical Intra-ocular Pressure

mu = 17
sigma = 2.5

Interocular Pressure (mmHg)

Density

Diagnose Unusual Intra-ocular Pressure

Task: Identify unusually low and high eye pressures for follow-up examination/treatment
What pressures are cutoffs for lower and upper 5% of distribution?

mu = 17
sigma = 2.5

Interocular Pressure (mmHg)
What z score cuts off top & bottom 5% of scores?

\[ z_U = 1.645 \text{ cuts off top 5%} \]
\[ z_L = -1.645 \text{ cuts off bottom 5%} \]

Diagnose Unusual Intra-ocular Pressure

Task: Identify unusually low and high eye pressures for follow-up examination/treatment

What pressures are cutoffs for lower and upper 5% of distribution?

\[ \mu = 17 \]
\[ \sigma = 2.5 \]

\[ z = \frac{X - \mu}{\sigma} \]
\[ X = z \times \sigma + \mu \]

\[ X_U = (1.645 \times 2.5) + 17 = 21.11 \]
\[ X_L = (-1.645 \times 2.5) + 17 = 12.88 \]

90% of “normal” intraocular pressure measures are between 12.88 and 21.11
5% of “normal” intraocular pressure measures are below 12.88 and 5% are above 21.11

Using cutoffs of 12.88 and 21.11 to diagnose “unusual” intraocular pressure

H0: Intraocular Pressure is Normal
H1: Intraocular Pressure is Abnormal (recommend follow-up tests)

If pressure is between 12.88 & 21.11: Do not reject H0
If pressure is < 12.88: Reject H0
If pressure is > 21.11: Reject H0

What is the probability of rejecting H0 by mistake?

Significance Level = \( \alpha = 0.1 \) or 10%
\( \alpha = \) Type I error rate

90% of “normal” intraocular pressure measures are between 12.88 and 21.11
5% of “normal” intraocular pressure measures are below 12.88 and 5% are above 21.11
Diagnose Unusual Intra-ocular Pressure

What pressures cutoff lower and upper 2.5% of distribution?

\[ \mu = 17 \]
\[ \sigma = 2.5 \]
\[ z_U = 1.96 \text{ cuts off top } 2.5\% \]
\[ z_L = -1.96 \text{ cuts off bottom } 2.5\% \]
\[ Z = \frac{(X - \mu)}{\sigma} \]
\[ X = (z \times \sigma) + \mu \]
\[ X_U = (1.96 \times 2.5) + 17 = 21.9 \]
\[ X_L = (-1.96 \times 2.5) + 17 = 12.1 \]

Significance Level = \( \alpha = 0.05 \) or 5%

\( \alpha \) = Type I error rate

95% of “normal” intraocular pressure measures are between 12.1 and 21.9
2.5% of “normal” intraocular pressure measures are below 12.1 and 2.5% are above 21.9

Importance of significance level (\( \alpha \))

- null hypothesis testing requires us to define criteria for distinguishing "unusual" scores (given null hypothesis is true)
- \( \alpha \) determines how extreme a score must be (given H0) before declaring it as “unusual” and, consequently, rejecting H0
- \( \alpha \) is the Type I error rate:
  - probability of rejecting H0 when H0 is, in fact, true
  - lowering \( \alpha \) (e.g., \( \alpha = .001 \) vs. \( \alpha = .05 \)) reduces Type I errors

Diagnose Unusually **High** Intra-ocular Pressure

Task: Focus on high inter-ocular pressure for early detection of glaucoma with alpha = 0.05

What value of pressure cuts off top 5%?

\[ z_{\text{High}} = 1.654 \]

cuts off approx top 5%

What z score cuts off top 5% of scores?

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Diagnose Unusually **High** Intra-ocular Pressure

**Task:** Focus on high inter-ocular pressure for early detection of glaucoma with alpha = 0.05

What value of pressure cuts off top 5%?

\[ \mu = 17 \]
\[ \sigma = 2.5 \]

**Z_high = 1.645**

\[ z = \frac{X - \mu}{\sigma} \]
\[ X = (z \times \sigma) + \mu \]

\[ X_{high} = (1.645 \times 2.5) + 17 = 21.1 \]

95% of “normal” intraocular pressure measures are less than 21.1
5% of “normal” intraocular pressure measures are above 21.1

Diagnose Unusually **High** Intra-ocular Pressure

**Using cutoff of 21.1 to define “high” intraocular pressure**

**H0:** Intraocular Pressure is Normal (< 21.1)
**H1:** Intraocular Pressure is High (≥21.1)

If pressure is ≤ 21.1: Do not reject H0
If pressure is > 21.1: Reject H0

What is the probability of rejecting H0 by mistake?

**Significance Level = α (one-tailed)= 0.05 or 5%**

α (one-tailed) = Type I error rate

95% of “normal” intraocular pressure measures are less than 21.1
5% of “normal” intraocular pressure measures are above 21.1